$$=\frac{2\pi}{n}\sqrt{4r^2\sin^2\frac{\pi}{n}+\frac{d^2}{n^2}}\frac{mn(mn-1)}{2}=\pi m(mn-1)\sqrt{4r^2\sin^2\frac{\pi}{n}+\frac{d^2}{n^2}}$$

$=\pi m^2\sqrt{4\pi^2r^4+d^2}$,when n is infinite but $r=1$ and $m=20$, $d=1$.

$\therefore S=400\pi\sqrt{(4\pi^2+1)}=7995.12$ feet approximately.

[The above revised solution of Professor Zerr's now agrees in result with that of Professor Hume previously published.—Editor.]

17. Proposed by H. W. DRAUGHON, Clinton, Louisiana

Find the volume generated by revolving a circular segment whose base is a given chord ($2a$), about any diameter as an axis.

II. First Solution by F. P MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy, New Windsor College, New Windsor, Maryland.

Let $\angle ECH=A$, $CA=r$, $AE=a$, $CE=x$, and $\angle ACD=\theta$; then $x'=r\cos\theta=\sqrt{(r^2-a^2)}$, and the required volume generated becomes

$$V=2(2\pi x\sin A)\int_{x'}^{r}\sqrt{(r^2-x^2)}dx=4\pi\sin A\left[-\tfrac{1}{3}\left(r^2-x^2\right)^{\frac{3}{2}}\right]_{x'}^{r}=\tfrac{4}{3}\pi a^3\sin A.$$

SECOND SOLUTION.

Put $\angle CAE=\phi$; then $x=r\sin\phi$, $dx=r\cos\phi d\phi$,and $\sqrt{(r^2-x^2)}=r\cos\phi$. The limits of integration are $\frac{1}{2}\pi$ and $\cos^{-1}(a/r)=\phi'$; and after obvious transformations, we have

$$V=4\pi\sin A\int_{\phi'}^{\frac{1}{2}\pi}r^3\cos^2\phi\sin\phi d\phi=4\pi\sin A\left[-\tfrac{1}{3}r^3\cos^3\phi\right]_{\phi'}^{\frac{1}{2}\pi}=\tfrac{4}{3}\pi a^3\sin A.$$

THIRD SOLUTION.

Represent the center of gravity of the segment $ADBE$ by G; then from Mechanics, we have $CG=\dfrac{\frac{1}{12}(2a)^3}{r^2\sin^{-1}(a/r)-a\sqrt{(r^2-a^2)}}$, in which the denominator is the area of the segment $ADBE,=\mathbf{A}$. Since $FG=CG\sin A=(2a^3/3\mathbf{A})\sin A$, the *Centrobaryc Method* gives for the volume generated $V=2\pi(2a^3/3\mathbf{A})\sin A\times\mathbf{A}=\frac{4}{3}\pi a^3\sin A$.

[In the solution by the Proposer,previously published the lower integral should have been $\sqrt{(R^2-a^2)}$. There was a misprint.—Editor.]

Note on Prob. No. 13 by JOHN DOLMAN, Jr., Philadelphia, Pennsylvania.

"It may be worth noting that the solution of No. 13 is incomplete in failing to state that the value of x found makes y a minimum only when angle EAD equals or is less than $\cos^{-1}\frac{4}{5}$ [considering the angle negative when D is west of E]. In all other cases the value, $x=\sin^{-1}\frac{4}{5}$, makes y a maximum and the steamers could *meet* in two possible points. All this, together with the critical value found, may be seen from inspection without any analytical work."

PROBLEMS.

27. Proposed by G. B. M ZERR. A. M. Principal of Schools, Staunton, Virginia.

A runs around the circumference of a circular field with velocity m feet; B starts from the centre with velocity $n>m$ feet to catch A. The straight line joining their positions always passes through the centre. Find the equation to the curve described by B, the distance he runs and the time occupied.